<u>Theorem</u>. Any reduced cohomology theory $\tilde{h}^*$ on pointed CW complexes such that $h^*(pt) = \tilde{h}^*(S^0) = \begin{cases} G & * = 0, \\ 0 & \text{otherwise} \end{cases}$ is isomorphic to $\tilde{H}^*(-; G)$.

<u>Cor</u>. $T : [X, K(G,n)]_* \xrightarrow{\cong} \tilde{H}^n(X; G)$, where $T([f]) = f^*(\alpha)$.

<u>proof</u>. Since this is a natural isomorphism, let $id : K(G,n) \to K(G,n)$ be represented by $\alpha \in H^n(K(G,n); G)$. Then, $T(id) = \alpha$.
Since $T(f) = T(f^*(id)) = f^*(T(id)) = f^*(\alpha)$, we're done.

<u>Consequences</u>. (1) If $x \in H^1(X, \mathbb{Z})$, then $x^2 = 0$.

(2)
$$\begin{tikzcd} X \arrow[r] & X \times X \arrow[r] & K(\mathbb{Z},m) \times K(\mathbb{Z},n) \arrow[r] & K(\mathbb{Z},m) \wedge K(\mathbb{Z},n) \arrow[r, "\text{Künneth}", "\text{Hurewicz}"'] & K(\mathbb{Z},m+n) \\ & & S^m \times S^n \arrow[u] \arrow[r] & S^m \wedge S^n \arrow[u] \arrow[ur] & \end{tikzcd}$$

Graded commutativity comes from the fact that $\tau : S^m \wedge S^n \to S^n \wedge S^m$ is degree $(-1)^{mn}$ on S^{m+n}.

(3) Postnikov obstructions.

$$\begin{tikzcd}
H^{n-1}(X^{n-1}) \arrow[d] & & \\
H^n(X^n, X^{n-1}) \arrow[rr, dashed] \arrow[d] & & H^{n+1}(X^{n+1}, X^n) \\
H^n(X^n) \arrow[urr] & &
\end{tikzcd}$$

dual of obvious map in homology.

proof. Cellular cohomology (unreduced) with coefficients in G can be computed as the cohomology of the chain complex

$$\cdots \to H^{n-1}(X^{n-1}, X^{n-2}; G) \to H^n(X^n, X^{n-1}; G) \to H^{n+1}(X^{n+1}, X^n; G) \to \cdots$$

where the maps are boundary maps for the pair (X^{n+1}, X^n).

Since X is a CW complex,

$$H^n(X^n, X^{n-1}; G) \underset{n>0}{\cong} \tilde{H}^n(X^n/X^{n-1}; G) \cong \tilde{H}^n(\bigvee_\alpha S^n_\alpha; G)$$
$$\cong \prod \tilde{H}^n(S^n_\alpha; G)$$
$$\cong \prod \tilde{H}^0(S^0; G).$$

This also works for $\tilde{h}^*$. The main question is whether the boundary maps are the same. This splits into two steps: First check that a degree m map induces multiplication by m on $\tilde{h}^n(S^n)$. This follows directly from the Eckmann-Hilton argument for the case of interest: $[S^n, K(G,n)]_*$ for $n \geq 1$. Argument in Hatcher for general case.

The second step is to worry about infinite coproducts. Namely, maps $\prod_\alpha G_\alpha \to \prod_\beta G_\beta$ are not necessarily determined by their restrictions to each G_α. However, this will be OK as each cell meets only finitely many parts of the product.