

Theorem. Any reduced cohomology theory  $\tilde{h}^*$  on pointed CW complexes such that  $\tilde{h}^*(pt) = \tilde{h}^*(S^0) = \begin{cases} G & * = 0 \\ 0 & \text{otherwise} \end{cases}$  is isomorphic to  $\tilde{H}^*(-; G)$ .

Cor.  $\tau: [X, K(G, n)]_+ \cong \tilde{H}^n(X; G)$ , where  $T(F) = F^+(\alpha)$ .

Proof. Since this is a natural isomorphism, let  $\text{id} \square: K(G, n) \rightarrow K(G, n)$  be represented by  $\alpha \in H^n(K(G, n); G)$ . Then,  $T(\text{id}) = \alpha$ . Since  $\square$   $T(F) = T(F^+(\text{id})) = F^+(T(\text{id})) = F^+(\alpha)$ , we're done.

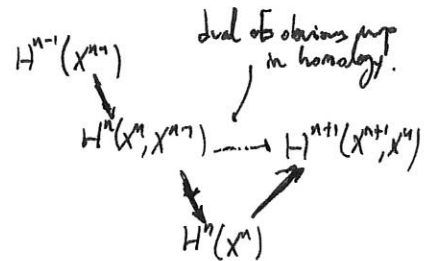
Consequences. (1) If  $x \in H^1(X, \mathbb{Z})$ , then  $x^2 = 0$ .

$$(2) \begin{array}{ccccccc} X \rightarrow X \times X & \longrightarrow & K(\mathbb{Z}_m) \times K(\mathbb{Z}_n) & \longrightarrow & K(\mathbb{Z}_m) \wedge K(\mathbb{Z}_n) & \xrightarrow[\text{Hurewicz}]{\text{Kunnet}} & K(\mathbb{Z}_m \wedge \mathbb{Z}_n) \\ & & \uparrow & & \uparrow & \nearrow & \\ & & S^m \times S^n & \longrightarrow & S^m \wedge S^n & & \end{array}$$

Graded commutativity comes from the fact that  $\tau: S^m \wedge S^n \rightarrow S^n \wedge S^m$  is degree  $(-1)^{mn}$  on  $S^{m+n}$ .

(3) Postnikov obstructions.

proof. Cellular cohomology<sup>1</sup> with coefficients in  $G$  can be computed as the cohomology of the chain complex



$$\dots \rightarrow H^{n-1}(X^{n-1}, X^{n-2}; G) \rightarrow H^n(X^n, X^{n-1}; G) \rightarrow H^{n+1}(X^{n+1}, X^n; G) \rightarrow \dots$$

where the maps are boundary maps for the pair  $(X^m, X^{n-1})$ .

~~$$H^n(X^m, X^{n-1}; G) \cong \tilde{H}^n(X^m/X^{n-1}; G) \cong \tilde{H}^n(S^n; G)$$~~

Since  $X$  is a CW complex,

$$\begin{aligned}
 H^n(X^m, X^{n-1}; G) &\cong \tilde{H}^n(X^m/X^{n-1}; G) \cong \tilde{H}^n(\bigcup_{\alpha} S^n_{\alpha}; G) \\
 &\cong \prod \tilde{H}^n(S^n_{\alpha}; G) \\
 &\cong \prod \tilde{H}^0(S^0; G).
 \end{aligned}$$

This also works for  $\tilde{H}^*$ . The main question is whether the boundary maps are the same. This splits into two steps: First check that a degree  $n$  map induces multiplication by  $n$  on  $\tilde{H}^n(S^n)$ . This follows directly from the Eckman-Hilton argument for the case of interest:  $[S^n, K(G, n)]_+$  for  $n \geq 1$ . Argument in Hatcher for general case.

The second step is to worry about infinite coproducts. Namely, maps  $\prod_{\alpha} G_{\alpha} \rightarrow \prod_{\beta} G_{\beta}$  are not necessarily determined by their restrictions to each  $G_{\alpha}$ . However, this will be OK as each cell meets only finitely many parts of the product.