

# 061 - Final - Practice Problems

1 June 2011

1. Prove that  $n! > n^2$  for all integers  $n \geq 4$ .
2. Let  $X$  be a finite set with  $n$  elements. Determine, with proof, how many binary equivalence relations there are on  $X$ .
3. How many rearrangements of MATHEMATICS are there where the Ms are not next to each other?
4. Let's play Canasta! The deck consists of 2 standard packs of 52 cards, 13 in each of 4 suits. So, there are 2 of every card, but we can't tell the two copies apart. For example, there are 2 Aces of Hearts. How many different 5-card hands are there that contain only Hearts?
5. Let  $X = \{1, 2, 3, 4, 5\}$ . How many strings of length 1000 on  $X$  are there such that there are no substrings from  $\{1, 2\}$  of length more than 1.
6. Prove that in any set of 51 positive integers less than 100, there are two whose sum is 100.

7. Show that if  $G$  is a simple graph, then either  $G$  or  $\overline{G}$  is connected.
8. Show that if  $G$  is a simple graph with at least two vertices, then there are two vertices in  $G$  with the same degree.
9. Prove that every tree with at least two vertices is a bipartite graph.
10. Prove that the number of nonisomorphic binary trees with  $n$  vertices is the  $n$ th Catalan number.