115b/1 - Practice Midterm

31 January 2010

1. Let $T : V \to V$ be a linear operator on a vector space V, and let $W \subseteq V$ be a T-invariant subspace. Suppose that $\lambda_1, \ldots, \lambda_k$ are distinct eigenvalues of T and that $v_1, \ldots, v_k$ are vectors such that $T(v_i) = \lambda_i v_i$ for $i = 1, \ldots, k$. Prove that if $v_1 + \cdots v_k$ is in W, then v_i is in W for $i = 1, \ldots, k$.

2. Let $T : V \to V$ be a linear operator on a finite dimensional vector space V, and let $W \subseteq V$ be a T-invariant subspace. Prove, using the result of problem 1, that if T is diagonalizable, then so is the restriction of T to W: $T|_W : W \to W$.

3. Let V be a finite dimensional real or complex inner product space. Show that if $T : V \to V$ is a normal linear operator, and if W is a T-invariant subspace of V, then $W^\perp$ is T^*-invariant.

4. Suppose that $T : V \to V$ is a linear operator on an n dimensional vector space such that V is a T-cyclic subspace of itself. Show that the minimal polynomial $p(t)$ of T has the same degree as the characteristic polynomial $f(t)$ of T.

5. Let $T : V \to V$ be a diagonalizable linear operator on a finite dimensional vector space V. Let $T^t : V^* \to V^*$ be the transpose of V. Show that T^t is diagonalizable.