115b/1 - Homework 4*

Due 31 January 2011

For any problem involving inner products, you may assume that the underlying field is either the real numbers or the complex numbers.

1. Find an example of a normal matrix over the real numbers that is not diaganolizable. You should prove both that your candidate matrix is normal and that it is not diagonalizable.

2. Prove that a normal matrix is self-adjoint if and only if its spectrum consists of real numbers.

3. Let A be an $n \times n$ matrix over $\mathbb{C}$, and suppose that for any $n \times n$ matrix X such that $tr(X) = 0$, we have $tr(AX) = 0$. Prove that $A = \lambda I_n$ [Prasolov].

4. Do problem (6.4.15).

5. A quadratic form $q : V \to F$ over a field is called anisotropic if it has no non-trivial zeros, and it is called isotropic if it has non-trivial zeros. The dimension of the quadratic form is the dimension of V by definition. A zero of quadratic form is an element v of V such that $q(v) = 0$. Such a zero is called trivial if $v = 0$. Show that any quadratic form of dimension at least 2 over $\mathbb{C}$ is isotropic. (See section 6.8 for material on quadratic forms.)

6. Construct anisotropic quadratic forms over $\mathbb{R}$ of any dimension $n > 0$.

7. Consider the vector space $V = \mathbb{C}^{2n}$ with basis $\{x_1, \ldots, x_n, y_1, \ldots, y_n\}$. On V, let q be the quadratic form $x_1y_1 + \cdots + x_ny_n$. Find an n-dimensional subspace of V on which q vanishes identically.

8. Do problem (6.2.18).

9. Do problem (6.4.13).

*Numbers in parentheses like (1.2.11) refer to the 11th problem in the second section of the first chapter of Friedberg *et. al.*, while [Prasolov] refers to the book *Problems and Theorems in Linear Algebra* by Prasolov.