115a/4 - Practice Midterm 2

5 November 2010

1. Let $T : P_2(\mathbb{R}) \to P_2(\mathbb{R})$ be the linear operator that takes a polynomial $f(x)$ to

$$f''(x)(x+2) + f'(x) + f(x).$$

Compute the characteristic polynomial of T. Show that T is *not* diagonalizable over $\mathbb{R}$.

2. Show that a linear operator on a finite-dimensional vector space is invertible if and only if it has zero nullity.

3. Say that a linear operator $T : V \to V$ is nilpotent if some n-fold composition T^n is the zero operator on V. That is, $T^n(v) = 0$ for all $v \in V$. Show that $\frac{d}{dx}$ is a nilpotent operator on $P_n(\mathbb{R})$ for any non-negative integer n.

4. Let $\beta = \{1, x, x^2\}$ be the standard ordered basis for $P_2(\mathbb{R})$, and let

$$\gamma = \{x^2 + x + 1, x, x^2 + 2\}.$$

This is another ordered basis for $P_2(\mathbb{R})$. Compute the change of basis (change of coordinate) matrices from β to γ and from γ to β. In the notation of class, compute $[Id]_\beta^\gamma$ and $[Id]_\gamma^\beta$.

5. Does the matrix

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

have any eigenvalues viewed as a linear transformation $\mathbb{Q}^2 \to \mathbb{Q}^2$? Prove your answer. Show that as a linear transformation $\mathbb{C}^2 \to \mathbb{C}^2$, A is diagonalizable.