

Cor. Let X be a $K(A, n)$ for $n \geq 1$. Then,

$$H_n(X, \mathbb{Z}) \cong A,$$

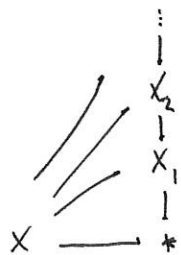
$$H^n(X, \mathbb{Z}) \cong \text{Hom}(A, \mathbb{Z}).$$

proof. This follows immediately from Hurewicz.

Co-building blocks feat. PT.

Let X be a topological space, path connected for simplicity.

Consider the process: kill the homotopy groups of X above degree i .



$$\text{So, } \pi_k X_i \cong \begin{cases} \pi_k X & k \leq i \\ 0 & k > i \end{cases}, \quad \text{and } \pi_k X \twoheadrightarrow \pi_k X_i$$

is an iso for $0 \leq k \leq i$.

Q. What is the homotopy fiber of $X_i \rightarrow X_{i-1}$?

The tower $\{X_i\}$ is called the Postnikov tower of X . We see that X is co-built from Eilenberg-MacLane spaces.

Thm. $[X, K(A, n)]_* \cong H^n(X, A)$.

sketch. $H^n(K(A, n), A) \cong \text{Hom}(H_n(A), A)$
 \parallel
 $\pi_n K(A, n)$.

Set $\alpha \in H^n(K(A, n), A)$ the image of Hurewicz.

Given $X \xrightarrow{f} K(A, n)$, get $f^*(\alpha) \in H^n(X, A)$.

Idea: it's true for $X = S^{n+k}$. So, it should be true in general.

Orientability. ~~$H^1(BSO_n, \mathbb{Z}/2) \cong \mathbb{Z}/2$~~

$$BSO_n \rightarrow B\mathbb{Z}/2 \cong K(\mathbb{Z}/2, 1)$$

\uparrow

fibr of $BSO_n \rightarrow (BSO_n)_1 \cong B\mathbb{Z}/2$.

The first Stiefel-Whitney class: $\omega_1(\xi) \in H^1(X, \mathbb{Z}/2)$

the obstruction to orientability.