Def. S pointed set,

G group,

A abelian group.

Eilenberg-MacLane spaces.

A $K(A,n)$-space is a pointed top. space X with $\pi_i(X) \cong \begin{cases} A & i=n, \\ 0 & \text{otherwise}. \end{cases}$

similarly for a $K(G,1)$ and $K(S,0)$.

Proposition. Eilenberg-MacLanes exist.

proof. $\coprod_{s \in S} *$ constructs $K(S,0)$.

Q: how to construct $K(G,1)$?

Pick generators α for A as an ab. grp. Set

$$X^n = \bigvee_\alpha S^n_\alpha.$$

So, $\pi_i(X^n) = 0$ for $i<n$, and $\pi_n(X^n) \cong \bigoplus_\alpha \mathbb{Z}$. Attach cells to kill relations. Get X^{n+1} with

$$\pi_i(X^{n+1}) = 0 \quad i<n,$$
$$\pi_n(X^{n+1}) = A.$$

Now, kill off all higher homotopy groups one dim at a time.

Remember: $\pi_i(X^k) \to \pi_i(X)$ is an iso for $i<k$ and a surjection for $i=k$.

Proposition. Any two $K(A,n)$ spaces X, Y are whe.

proof. It is enough to show that any one is whe to the standard one we constructed. So, say Y is a $K(A,n)$,

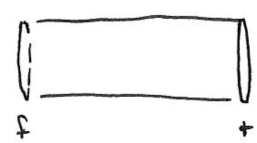
$$X^n \subset X^{n+1} \subset \cdots \quad \subset X$$

is the standard one. Choice of gens of A induces

$$\begin{tikzcd}
 & & X^n \arrow[r] \arrow[d, phantom, "\cap"] & Y \\
S^n \arrow[urr, "\phi_\alpha"] \arrow[r, phantom, "\subseteq"] & D^{n+1} \arrow[r, phantom, "\subseteq"] & X^{n+1} \arrow[ur, dashed] \arrow[d, phantom, "\cap"] & \\
 & & X^{n+2} \arrow[uur] & \\
 & & \vdots & \\
 & & X \arrow[uuuur] &
\end{tikzcd}$$

$\longleftarrow$ these should be dashed.

A map $f: S^n \to Y$ is nullhomotopic iff it can be filled in to $D^n \to Y$. Indeed, consider



$\simeq D^n$.

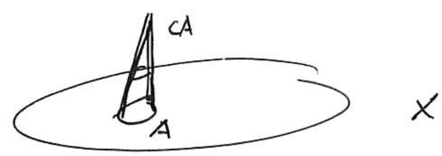
Cor. Any two CW $K(A,n)$s are he.

<u>Proposition</u>. A CW pair (X,A) is r-connected, and A is s-connected. Then,

$$\pi_i(X,A) \to \pi_i(X/A)$$

is an iso for $i \le r+s$ and a surjection for $i = r+s+1$.

<u>proof</u>. Look at $X \cup CA$.



Now, $CA \subset X \cup CA$ is contractible. So, $X \cup CA \to (X \cup CA)/CA \simeq X/A$ is a h.e.

$$\begin{tikzcd}
\pi_i(X,A) \arrow[r] & \pi_i(X \cup CA, CA) \arrow[r] & \pi_i(X \cup CA/CA) \cong \pi_i(X/A) \\
 & \pi_i(X \cup CA) \arrow[u, "\wr"', "LES"] \arrow[ur, no head, "\simeq"] &
\end{tikzcd}$$

Since (CA,A) is $(s+1)$-connected, excision (BME) applies to the first arrow.