313 - HW6

Due in class 5 December 2014

Please give a full definition or statement of any italicized terms appearing below.

1. Prove that $\sqrt{18}$ is *irrational* both directly and using the *rational zeros theorem.*

2. Use the *completeness axiom* to show that there is a real solution to $x^2 - 18 = 0$.

3. Prove that if $r < s$ are rational numbers there exists an *irrational number* x such that $r < x < s$.

4. Let x_n be a *sequence* of real numbers that *converges* to x. Suppose that $x_n \in [a, b]$ for all n. Prove that $x \in [a, b]$.

5. Prove that $\lim_{n\to\infty} n^{1/n} = 1$.

6. Prove that $\lim_{n\to\infty} \sqrt{n}x^n = 0$ for $|x| < 1$.

7. Prove that any convergent sequence is *Cauchy.*

8. Prove that *bounded monotonic* sequences converge.

9. Prove that every sequence has a monotonic *subsequence.*

10. Prove the *Bolzano-Weierstrass theorem.*

11. Use the *comparison test* and *geometric series* to prove that the series $\sum_{n=1}^{\infty} a_n$ *converges absolutely* if $\limsup |a_n|^{1/n} < 1$.

12. Prove that $\mathbb{R}$ is *uncountable.*

13. Prove that if $f : [a, b] \to \mathbb{R}$ is *continuous*, then f is *uniformly continuous.*

14. Show that if $f_n : [a, b] \to \mathbb{R}$ is a sequence of continuous functions *converging uniformly* to f, then f is continuous.

15. Give a counterexample to the previous problem when the sequence only *converges pointwise.*

16. Prove that a continuous function $f : [a, b] \to \mathbb{R}$ is *Darboux integrable.*

17. Use the *mean value theorem* to prove the *first fundamental theorem of calculus.*