308G - Quiz 2

25 April 2014

Name:
Student ID #:

This is a closed-book, closed-notes quiz. Calculators are not allowed.

1. (1 point). Is the matrix

$$A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 4 & 0 \\ 0 & 1 & 0 & 6 \end{pmatrix}$$

singular?

NO. If we look at one of many things we can determine that this matrix is non-singular. Linearly independent column vectors, non-zero determinant, invertible. All of these conditions are equivalent to non-singular.

2. (1 point). Let $\mathbf{A}_1$, $\mathbf{A}_2$, $\mathbf{A}_3$, and $\mathbf{A}_4$ be the columns of the matrix displayed above. Are they linearly dependent?

NO. In fact the answer to the previous question is an equivalent statement to this. However if we look at any one of the column vectors, and try to write it as a combination of the remaining column vectors, we will see that it is not possible.

3. (2 points). Let

$$B = \begin{pmatrix} 2 & 3 \\ 3 & 5 \end{pmatrix}.$$

This matrix is invertible with inverse

$$B^{-1} = \begin{pmatrix} 5 & -3 \\ -3 & 2 \end{pmatrix}.$$

Find the unique solution to $B\mathbf{x} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

For this we just solve the following equation;

$$B^{-1}B\mathbf{x} = \mathbf{x} = B^{-1}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\mathbf{x} = \begin{pmatrix} 5 & -3 \\ -3 & 2 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$$

4. (1 point). Suppose that C is invertible. Then, we know that C^T, the transpose of A, is also invertible. Is its inverse C^{-1} or $(C^{-1})^T$? (Circle one.)

$(C^{-1})^T = (C^T)^{-1}$

5. (5 points). Find the inverse of the matrix

$$D = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 3 & 4 & 1 \end{pmatrix}.$$

Show all work for this problem.

First augment with the identity matrix, then use elemetary row operations to produce the identity matrix on the other side.

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 3 & 4 & 1 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_2-2R_1} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -2 & -2 & 1 & 0 \\ 3 & 4 & 1 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_3-3R_1} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -2 & -2 & 1 & 0 \\ 0 & 1 & -2 & -3 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{R_3+R_2} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -2 & -2 & 1 & 0 \\ 0 & 0 & -4 & -5 & 1 & 1 \end{bmatrix} \xrightarrow{(-1/4)*R_3} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -2 & -2 & 1 & 0 \\ 0 & 0 & 1 & 5/4 & -1/4 & -1/4 \end{bmatrix}$$

$$\xrightarrow{(-1)*R_2-2R_3} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1/2 & -1/2 & 1/2 \\ 0 & 0 & 1 & 5/4 & -1/4 & -1/4 \end{bmatrix} \xrightarrow{R_1+R_2+R_3} \begin{bmatrix} 1 & 0 & 0 & 1/4 & 3/4 & -1/4 \\ 0 & 1 & 0 & -1/2 & -1/2 & 1/2 \\ 0 & 0 & 1 & 5/4 & -1/4 & -1/4 \end{bmatrix}$$

Thus the required matrix is

$$D^{-1} = \frac{1}{4}\begin{pmatrix} 1 & 3 & -1 \\ -2 & -2 & 2 \\ 5 & -1 & -1 \end{pmatrix}$$