

Homework 6 - Math 132/3

Due 25 May 2012

1. Discuss the convergence and uniform convergence of the infinite series

$$\sum_{n=1}^{\infty} nz^n.$$

2. Where does the series

$$\sum_{n=1}^{\infty} \frac{x}{n(1+nx^2)}$$

converge uniformly, where x denotes a real number?

3. If $f(z)$ is a function defined by the power series

$$f(z) = \sum_{n=0}^{\infty} a_n z^n,$$

what function is

$$\sum_{n=0}^{\infty} n^3 a_n z^n?$$

4. Suppose that the power series

$$\sum_{n=0}^{\infty} a_n z^n$$

has radius of convergence equal to R . What are the radii of convergence of the series

$$\sum_{n=0}^{\infty} a_n z^{2n}$$

and

$$\sum_{n=0}^{\infty} a_n^2 z^n?$$

5. Compute the Taylor series for $f(z) = e^z$ centered at $2\pi i$. Compute the Taylor series for $\cosh z$ centered at $z = 2\pi i$. Compute the Taylor series for $z^5 e^z$ centered at $z = 0$.