Math 61
Midterm I
February 6, 2007

Name:____________________, ____________________
Please put your last name first and print clearly

Signature:______________________________

TA section you are attending (Tues or Thurs):_____

1.__________

2.__________

3.__________

4.__________

5.__________

Total__________

You must put all your answers in the spaces provided on the page of the problem . **Please do not use the spaces on this page.** You must show a method of solution to obtain credit for a problem. You need not simplify your answers and you can leave your answers in terms of C(n,r)=$\binom{n}{r}$ or P(n,r).

NO CALCULATORS!

1. Show using induction that the statement $P(n)$:

$$2(1 + 3 + 3^2 + \ldots + 3^{n-1}) = 3^n - 1$$

is true for $n \geq 1$ showing all your steps and using compete sentences.

2. Let

$$X = \{1, 2, 3, 4\}$$

and let

$$Y = \{5, 6, 7, 8\}$$

a) Give an example of a function $f : X \to Y$ which is not one-to-one. Specify f by writing out elements of $X \times Y$.

b) Explain in a complete sentence why your function is not one-to-one.

c) What is the range of your function?

3a. Let

$$X = \{0, 1, 2, 3, 4\}$$

and define an equivalence relation R on X by

$$n\,R\,m \quad \text{if } n^2 - m^2 \text{ is divisible by } 5$$

(You need not show that this is an equivalence relation.) List all the **distinct** equivalence classes of the relation R.How many equivalence classes are there?

3b. Let

$$Y = \{0, 1, 2, 3, 4\}$$

Give an example of a relation on Y which is reflexive and symmetric, but not transitive. You should do this by listing the elements of $Y \times Y$ which make up the relation. Explain in complete sentences why your relation is not transitive, but you need not be longwinded. You need not show it is reflexive or symmetric. This problem is unrelated to 3a.

4a. How many solutions to

$$x_1 + x_2 + x_3 + x_4 = 14$$

are there with the $x_i \geq 0$ and the x_i integers? You should explain a method to calculate the answer, not just apply a formula.

4b.How many solutions to

$$x_1 + x_2 + x_3 + x_4 = 14$$

are there with the $6 \geq x_i \geq 1$ and the x_i integers?

5. How many seven card hands are there from an ordinary deck of 52 cards with 3 of one suit, two of a different suit and two of yet a different suit? (So three suits will be represented in the hand. There are four suits in the deck, each with 13 cards.)